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1. [1 PT each] Say whether the following statements are true or false:

1) Checking the correctness of a language means checking its semantics	□ **F**
2) ULM and Entity-Relationship (ER) models have formal syntax	□ **T**
3) In the extensional semantics each symbol is assigned an object of the domain	□ **F**
4) Given a formal language, a model is an interpretation function	□ **T**
5) Model checking is the service that finds a model satisfying a given proposition P	□ **F**
6) In PL, a proposition P is unsatisfiable iff its negation is satisfiable	□ **F**
7) DPLL procedure is always polynomial in time	□ **F**
8) In DPLL, the pure literal step assign a truth value to all the symbols appearing once in the formula	□ **F**
9) The PL formula A → A is valid	□ **T**
10) Instance checking id the ABox service that finds all the instances of a concept C	□ **F**

2. [2 PT] Show how to reduce in ClassL unsatisfiability and disjointness to subsumption.

C unsat iff $C \sqsubseteq \bot$

$C \perp B$ iff $C \sqcap B \sqsubseteq \bot$

3. [5 PT] Define for a formula P in ClassL what it means

- **a. to be true in a model**
- **b. to be satisfiable**
- **c. to be valid**
- **d. to be unsatisfiable**
- **e. define the relations between sat, unsat and valid**

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a) P is true in a model I iff I(P) is not the empty set.

b) P is sat iff there exist an I such that I(P) is not the empty set.

c) P is valid iff for all I then I(P) is not the empty set.

d) P is unsat iff there is not an I such that I(P) is not the empty set.

e) valid(P) iff unsat(¬P)

sat(P) ∧ ¬ valid(P) iff sat(¬ P) ∧ ¬ valid(¬ P)

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4. **[2 PT] Using a BNF grammar, provide the syntax of the PL language and then the corresponding intentional semantics.**

5. **[2 PT] Define a PL language and a theory for the following problem: "Managers are employees. A company is important if there is at least a manager". Provide also a model for the theory in which manager is false.**

L = {company, manager, important, employee}
T = {manager → employee, manager → company ∧ important}

I(manager) = F
I (employee) = T
I(company) = T
I(important) = F

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6. **[1 PT] Represent the following problem using ClassL (define a TBox and ABox if needed): "*Bananas can be yellow or red fruits. Monkeys eat only yellow bananas. Moogle is a monkey.*"**

$T = \{Banana \sqsubseteq Fruit \sqcap (Yellow \sqcup Red), Monkey \sqsubseteq EatBanana \sqcap Banana \sqcap Yellow\}$

$A = \{Monkey(Moogle)\}$

7. **[2 PT] Using the tableaux calculus, determine whether the PL formula $(C \rightarrow \neg B) \wedge B \wedge \neg(C \vee A)$ is valid.**

The formula can be rewritten as follows: $(\neg C \vee \neg B) \wedge B \wedge (\neg C \wedge \neg A)$
To prove that it is valid we need to prove that its negation is unsatisfiable, i.e. that all the branches are closed:

$\neg ((\neg C \vee \neg B) \wedge B \wedge (\neg C \wedge \neg A)) \equiv (C \wedge B) \vee \neg B \vee (C \vee A)$

The corresponding tableaux is therefore:

```
   (C ∧ B) ∨ ¬B ∨ (C ∨ A)
    /        |        \        \
   C        ¬B        C        A
   |
   B
```

where all the branches are open. As a consequence the formula is not valid.

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8. [2 PT] Given the TBox T = {B ⊑ A, B ⊑ C}, provide a model using Venn diagrams such that T ⊨ ¬A ⊓ C. Shadow the area corresponding to ¬A ⊓ C.

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9. [2 PT] Provide the steps and the output of the DPLL algorithm for the PL formula (C → A) ∧ (A → B) ∧ ¬B ∧ A

The formula has to be rewritten in CNF:

(¬C ∨ A) ∧ (¬A ∨ B) ∧ ¬B ∧ A

By assigning the truth values to the unit clauses I(B) = F and I(A) = T, and by propagating them we obtain that the first clause is eliminated while the second becomes an empty clause.

As a consequence the DPLL returns false.

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10. [2 PT] Given the TBox T above, prove by concept expansion that T $\models$ Assistant $\sqsubseteq$ Student

Undergraduate	**$\sqsubseteq \neg$ Teach**	**PhD**	**$\equiv$ Master $\sqcap$ Research**
Bachelor	**$\equiv$ Student $\sqcap$ Undergraduate**	**Assistant**	**$\equiv$ PhD $\sqcap$ Teach**
Master	**$\equiv$ Student $\sqcap \neg$ Undergraduate**		

Assistant $\equiv$ PhD $\sqcap$ Teach

PhD $\sqcap$ Teach $\equiv$ (Master $\sqcap$ Research) $\sqcap$ Teach

Master $\sqcap$ Research $\sqcap$ Teach $\equiv$ (Student $\sqcap \neg$ Undergraduate) $\sqcap$ Research $\sqcap$ Teach

(Student $\sqcap \neg$ Undergraduate) $\sqcap$ Research $\sqcap$ Teach $\sqsubseteq$ Student

Therefore Assistant $\sqsubseteq$ Student

11. [3 PT] Given the following TBox T and the ABox A:

T = { A $\sqsubseteq \neg$ B, C $\equiv$ D $\sqcap$ A , E $\sqsubseteq$ D } A = { C(a), E(c), B(b) }

(a) Provide the expansion of A w.r.t. T (without normalizing T)

C(a), D(a), A(a), ¬B(a),
E(c), D(c), B(b)

(b) Provide the instance retrieval of D

{a, c}

(c) Say (YES/NO) weather by adding ¬A(c) to the ABox it remains consistent or not w.r.t. T. Motivate your response.

Yes, because there is no assertion A(c) in the expanded ABox.